Lesson 39: Kinetic Energy & Potential Energy

Kinetic Energy   Work-Energy Theorem   Potential Energy

Total Mechanical Energy

We sometimes call the total energy of an object (potential and kinetic) the **total mechanical energy** of an object.

- "Mechanical" energy doesn't mean that it always has to involve machines.
    - An apple falling off a cliff has **gravitational potential** and **kinetic energy**, so it therefore has **mechanical energy**.
- We will start off by looking at the individual kinds of energy, and then at how we can start to join them together into one big idea.

Kinetic Energy

You've probably heard of **kinetic energy** in previous courses using the following definition and formula…

Any object that is moving has kinetic energy.

$$E_k = ½\, mv^2$$

E_k = kinetic energy (J)
m = mass (kg)
v = velocity (m/s)

We're going to keep on using that basic formula, but we do need to clear up the definition a little bit.

What is "any object"?

- "Any object" just refers to anything that we can measure as having a **mass**.
- This covers everything from small subatomic particles like electrons all the way up to galaxies.

When they say "moving" we need to ask "Moving relative to what?"

- Right now you're sitting motionless at a computer screen, so you have no kinetic energy, right?
    - This is true relative to the reference frame of the room you're in. Isn't the earth spinning on its axis? Isn't the whole planet moving around the sun?
- You need to make sure that you are always sure about what your measurements are being taken in relation to.
    - Most of the time we measure stuff relative to the surface of the earth, so things are easier, but be careful.

Example 1: A pop can with a mass of 312g is sitting in the cup holder of my car as I drive down Yellowhead at 68 km/h.

a) **Determine** how much kinetic energy it has relative to me in the car.

$E_k = ½ mv^2$

But relative to me the pop can's velocity is zero, so…

$E_k = 0$ J

b) **Determine** how much kinetic energy it has relative to someone standing on the side of the road.

$E_k = ½ mv^2$

$= ½ (0.312 \text{ kg}) (19 \text{ m/s})^2$

$E_k = 56$ J

Also, be ready to manipulate this formula to solve for other variables…

Example 2: Determine the velocity of a 150 kg cart if it has 3.60e4J of kinetic energy.

First, see if you can correctly solve the formula for "v". This is one of the manipulations that students commonly mix up! You should get…

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(3.60e4)}{150}} = 21.9\, m/s$$

The concept of kinetic energy can also come in handy if you need to perform calculations of the work done, as the following example shows…

Example 3: I am driving my 2500 kg Camaro down the street at 52 km/h. I notice that there is a school zone ahead, so I hit the brakes to slow down to 24 km/h. If I slowed down over a distance 145 m, **determine** the average force applied by the brakes.

First, you'll need to change those velocities from km/h into m/s…

$v_i = 52 \text{ km/h} = 14 \text{ m/s}$

$v_f = 24 \text{ km/h} = 6.7 \text{ m/s}$

Remember to keep the complete number (14.44444444) from your calculator written down on scrap paper…

Next, calculate the change in kinetic energy of the car before and after…

$E_k = ½ mv_i^2 = ½ (2500)(14)^2 = 260802$ J

$E_k' = ½ mv_f^2 = ½ (2500)(6.7)^2 = 55556$ J

$\Delta E_k = E_k' – E_k$

$\Delta E_k = -205246$ J

Since work is a change in energy $W = \Delta E_k$, but work also equals F d.

$\Delta E_k = W = Fd$

$\Delta E_k = Fd$

$F = \Delta E_k / d = (-205246) / (145m)$

$F = -1415 \text{ N} = -1.4e3 \text{ N}$

The negative sign just shows that the force is being exerted in the opposite direction to the velocity of the object.

Work-Energy Theorem

The fact that work is equal to the change in energy in a situation is usually called the "**Work-Energy Theorem**"

- This means that doing work on an object in some way changes the mechanical energy of the object.
- If you look at the last example, it makes sense...
    - The car changed its velocity, which means it accelerated.
    - Acceleration is caused by a force.
    - This force acted over a displacement.
    - Work happens when a force acts over a displacement.
- In many questions you can say that the works done causes a change in the kinetic energy, but it can also change the potential energy of the object.

$$W = \Delta E$$

$$W = \Delta E_k + \Delta E_p$$

Kinetic Energy Work-Energy Theorem Potential Energy

Potential Energy

Potential energy is another concept that you have already studied. You were probably told…

Potential energy is stored energy that is able to do work later.

This means that potential energy can be found in many different forms:

1. A car battery is storing energy in the chemical bonds between the substances inside. This is **chemical** potential energy.
2. An elastic that you stretch to shoot at a classmate across the room has energy stored in it as **elastic** potential energy.
3. Matter itself is actually energy in a stable, stored form. This is what Einstein's famous E = mc2 formula shows us, and is the basis for the release of energy from nuclear reactions like fission and fusion. It is sometimes called the **rest mass** potential energy.
4. An object held up in the air is a form of stored energy, since as soon as it is released it will start to fall. It could do work because of this release of energy. This particular kind of potential energy is the main topic of this lesson and is called **gravitational** potential energy.

Gravitational Potential Energy

We spend most of our time studying gravitational potential energy:

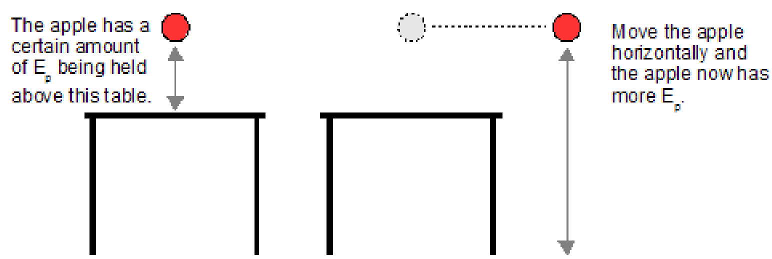
$$E_p = mgh$$

E_p = potential energy (J)
m = mass (kg)
g = gravity (m/s^2)
h = height (m)

In the formula watch out how you use the variables.

1. "m" is the mass of the object being held up. If an apple is being held above the ground, use the mass of the apple in your calculations, not the earth's mass.
2. "g" is the acceleration due to gravity. Since we will most often be doing questions that relate to earth, you can use 9.81m/s2, but every so often questions will appear that are not on earth, so make sure to use the appropriate value.
3. "h" is the height of the object above some reference point, which should be whatever the object will hit if it is dropped. If you're holding an apple above a table, don't use the earth as a reference point, use the table top.

The weird part is that you can change the gravitational potential energy of an object by moving it horizontally (no change in its actual vertical position) because of reference points.



- We are ***not*** creating energy when we do this.
- Instead, we are just changing the proportion of E_p compared to other forms of energy by changing the reference point we are using.
- In the first case we used the table as a reference point, and in the second we used the ground. If you change the reference point, then you must start at the beginning and recalculate everything.

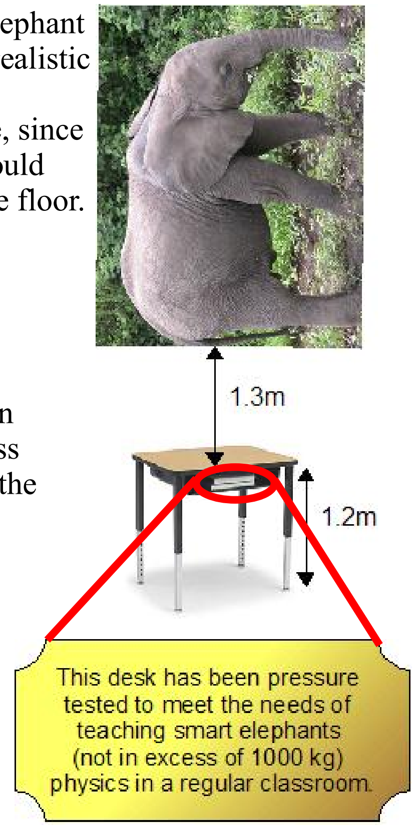
Example 4: **Determine** the gravitational potential energy of a 900 kg elephant held 1.3 m above a desk that is 1.2 m high, as shown in the completely realistic and undoctored photo to the right.

I think we should *not* use the desk top as the reference point here, since the chances of it supporting an elephant are slim. Instead, we should consider the elephant to be 2.5 m above the reference point of the floor.

$E_p = mgh$
$= (900)(9.81)(2.5)$
$E_p = 2.2e4$ J

If we were told that the desk is made out of titanium and has been verified by the manufacturer as being able to support the full mass of up to a 1000 kg elephant studying physics, then we could use the desk top as our reference point.

$E_p = mgh$
$= (900)(9.81)(1.3)$
$E_p = 1.1e4$ J



In this example you get two different (but both correct) answers for the gravitational potential energy of the elephant.

- This is simply because different reference points have been chosen.
- In most questions you won't have to try to figure out the reference point based on information like in this example. You will simply be given the height above a certain reference point and that is all there is to it.
  - You do ***not*** have to take into account any other distances other than the height.

Example 2: **Determine** how high above the ground the elephant (from the previous example) is if he has 500 J of gravitational potential energy.

$$E_p = mgh$$
$$h = \frac{E_p}{mg}$$
$$h = \frac{500}{900(9.81)}$$
$$h = 0.0566\,m$$

Elastic Potential Energy

Any object than can be ***deformed*** (have its shaped changed) and then return to its original shape can store elastic potential energy.

- We're still talking about potential energy, since it is stored energy until the object is allowed to bounce back.
- "Elastic" does not refer to just things like elastic bands…other materials that would be referred to as elastic would be...
  - pole vaulter's pole
  - springs
  - cheese (no, I'm just kidding. Just wanted to see if you're paying attention)

Robert Hooke, a man who can only be called a genius for his contributions to the early study of **biology** (he wrote and illustrated the Micrographia), **chemistry** (he helped Robert Boyle come up with Boyle's Law), made the contribution to **physics** of **Hooke's Law** for explaining the elasticity of objects.

$$F_s = -kx$$

F_s = force (N)
k = spring constant for that object (N/m)
x = amount of expansion (+) or compression (-) (m)

The reason for minus sign in the formula is that using the formula in this form is meant to calculate the amount of force trying to **restore** the spring back to its original position (it's usually called the **restoring force**, no surprise).

- If the spring has been stretched (which means it would have a positive "x" value), there will be a force trying to return it back the opposite direction (the force would be negative).

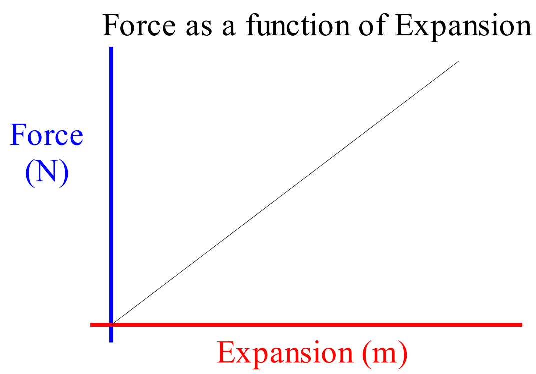
Example 3: If a spring has a spring constant of 18.5 N/m, **determine** the restoring force when **a)** the spring is expanded 15cm and **b)** compressed 20cm from its equilibrium point.

a) $F_s = -kx = -(18.5\text{N/m})\ (+0.15\text{m}) = -2.8\ \text{N}$
Since the spring was expanded the force acts to return it to its original position (the equilibrium point).

b) $F_s = -kx = -(18.5\text{N/m})\ (-0.20\text{m}) = +3.7\ \text{N}$
In this case the spring was compressed, so the restoring force tries to move it back the other direction.

We can use this formula to figure out a formula for the energy stored in the spring.

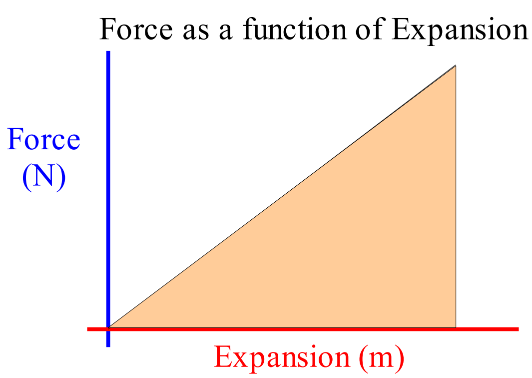
- Remember that W = F d
  - We might be tempted to just shove the formula for Hooke's Law into this formula to get something like $W = kxd = kx^2$, but this is wrong!
  - You have to take into account that the force is not constant as the object returns to its original shape… it's at a maximum when it is deformed the most, and is zero when the object is not deformed.
- Let's graph Force vs Distance of Expansion for a spring that we stretched…



The slope of this line (rise over run) would be Force over Expansion... the slope is equal to the spring constant.

Illustration 1: Graph for a stretched spring.

But this is really just a Force vs Displacement Graph like the ones we just looked at a couple lessons ago! To figure out the energy of the spring we can just figure out the work it does by looking at the area under the graph.



$$\text{Area} = \tfrac{1}{2}bh$$
$$= \tfrac{1}{2}Fx$$
$$= \tfrac{1}{2}(kx)x$$
$$\text{Area} = \tfrac{1}{2}kx^2 = W$$

$F = kx$
We ignore the negative sign in the formula since the resulting formula is for measuring energy, which is scalar.

Illustration 2: Area under the line.

So the work the spring will do to restore its shape (the energy it stored) can be calculated using…

$$E_p = \tfrac{1}{2}kx^2$$

E_p = eleastic potential energy (J)
k = spring constant (N/m)
x = amount of expansion or compression [deformation] (m)

Example 4: **Determine** how much energy a spring with a spring constant of 15 N/m stores if it is stretched by 1.6m.

$$E_p = \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}(15\text{N/m})(1.6\text{ m})^2$$
$$E_p = 19\text{ J}$$

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