Lesson 40: Conservation of Energy

A large number of questions you will do involve the t**otal mechanical energy** of a system.

- As pointed out earlier, the **mechanical energy** is just the total of all types of energy.
  - In many questions you will just have to concern yourself with kinetic and gravitational potential energy.
- Although there is no formal formula for **mechanical energy**, it could look something like...

$$E_m = E_k + E_p$$

Example 1: A 14 300 kg airplane is flying at an altitude of 497 m at a speed of 214 km/h. **Determine** the airplane's total mechanical energy.

Remember to convert the speed into metres per second.

$$E_m = E_k + E_p$$
$$E_m = \frac{1}{2}mv^2 + mgh$$
$$E_m = \frac{1}{2}(14300)(59.\bar{4})^2 + 14300(9.81)(497)$$
$$E_m = 9.50e7\,J$$

Notice that the calculation and the answer have nothing to do with direction, since energy is ***scalar***.

In many questions we assume that the objects we are dealing with are in an **isolated system**.

- An isolated system simply means a situation where *nothing*, not even energy, can enter or leave. Whatever you started with you finish with.
- There are three kinds of systems, and they are defined by whether or not matter and energy are conserved (remain constant).

System	Matter	Energy
Isolated	**Conserved**	**Conserved**
Closed	**Conserved**	**Not Conserved**
Open	**Not Conserved**	**Not Conserved**

To be an isolated system we must also have no outside forces acting on the objects.

- An outside force would mean there was the possibility of work being done, which would change the energy of the system.
- As long as we deal with isolated systems that no outside forces act on, we know that the total mechanical energy **before** and **after** will be equal.

$$E_m = E_m'$$
$$E_k + E_p = E_k' + E_p'$$

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- So if the one type of energy decreases, the other type of energy will increase by a similar amount.
    - Energy is not being created or changed, it is only changing forms or transferring from one object to another. This is known as the **Law of Conservation of Energy**.

Example 2: A person is sitting on a toboggan at the top of a 23.7m tall hill. If the person and toboggan have a total mass of 37.3 kg , **determine** how fast they will be going when they reach the bottom of the hill. Assume there is no friction.

- If there was friction, then it would not be an isolated system. The frictional force would result in some heat being given off, which would be energy leaving the system.
- At the top of the hill the person isn't moving, so E_k will be zero. At the bottom of the hill the E_p will be zero because the person is zero metres above the reference point.

$$E_k + E_p = E_k' + E_p'$$

$$\frac{1}{2} mv^2 + mgh = \frac{1}{2} mv'^2 + mgh'$$

$$0 + (37.3)(9.81)(23.7) = \frac{1}{2}(37.3) v'^2 + 0$$

$$8.67e3 = 18.7 v'^2$$

$$v' = 21.6 \text{ m/s}$$

Notice how in this example all of the potential energy the object had at the **top** of the hill has been turned completely into kinetic energy at the **bottom**.

- It's also possible to analyze how the potential energy steadily changes into kinetic energy during a fall…

Example 3: Wille E. Coyote is trying to drop a boulder off a cliff to hit the Roadrunner eating a bowl of birdseed. He wants to know the speed of the boulder at various points. He supplies you with the blueprint shown here…

The Coyote wants you to **determine** the velocity of the boulder at several different heights above the ground, assuming no air resistance…

a) 45 m
b) 30 m
c) 10 m
d) 0 m

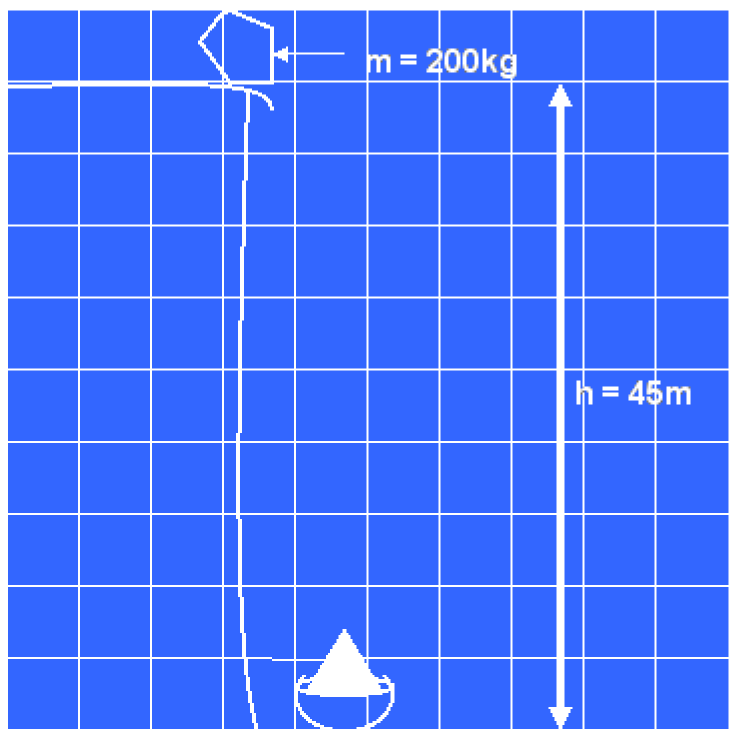

Illustration 1: The Coyote's Plan.

a) Well, this one ain't so tough! Since it's sitting at the top of the cliff, its velocity is 0 m/s. It might be handy at this point to calculate how much E_p the boulder has at this time, since this is also the total mechanical energy it starts out with!

E_p = mgh = 200(9.81) (45) = 88290 J = 8.8e4 J

TOTAL ENERGY = 8.8e4 J

b) First, ask yourself how much E_p the boulder still has at 30m above the ground.

E_p = mgh = 200 (9.81) (30) = 58860 J = **5.9e4 J**

That means that **8.8e4 J** - **5.9e4 J** = **2.9e4 J** is missing, right?

Wrong! According to the conservation of mechanical energy, that energy must now be **kinetic**!

$$E_k = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$v = \sqrt{\frac{2(2.9e4)}{200}}$$
$$v = 17\,m/s$$

So it's falling at **17m/s** at this time.
TOTAL ENERGY = 8.8e4 J

c) Again, calculate how much E_p you have at this new height of 10 m…

E_p = mgh = 200 (9.81) (10) = 19620 J = **2.0e4 J**

That means that I have changed **6.9e4 J** of energy into other forms… we'll assume it all changed into kinetic energy.

$$E_k = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$v = \sqrt{\frac{2(6.9e4)}{200}}$$
$$v = 26\,m/s$$

So it's falling at **26m/s** at this time.
TOTAL ENERGY = 8.8e4 J

d) By the time the boulder has reached the ground, all of its potential energy is gone (it's zero metres above the ground!). We all know that when it actually hits the ground it will come to rest, but we are concerned with how fast it's going when it is right at ground level but hasn't actually touched the ground yet. We can assume that all of the potential energy the boulder had at the top is now kinetic energy at the bottom…

$$E_k = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$v = \sqrt{\frac{2(8.8e4)}{200}}$$
$$v = 30 m/s$$

So it's falling at **30m/s** at this time.
TOTAL ENERGY = 8.8e4 J

You could be finding the same answers based on kinematics formulas from earlier lessons.

- In fact, you'll find that conservation of energy gives you new ways to do many problems that you did with kinematics formulas.

More Examples

Example 4: A toy car with a mass of 212g is pushed by a student along a track so that it is moving at 12m/s. It hits a spring (k = 52.8 N/m) at the end of the track, causing it to compress.

a) **Determine** how far did the spring compress to bring the car to a stop.
b) If the spring only compressed 50 cm in bringing the car to a stop, **explain** what happened.

a) We know that conservation of mechanical energy means that the kinetic energy of the car moving will be turned into elastic potential energy stored in the spring. So…

$E_k + E_p = E_k' + E_p'$
$E_k = E_p'$
$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$

$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{0.212(12)^2}{52.8}} = 0.76 m$$

b) If the spring didn't get pushed in as much, it must mean that some of the energy wasn't transferred perfectly… there must have been some waste heat released (not an isolated system). To calculate how much, first figure out how much elastic energy the spring ***did*** absorb…

$E_p = \frac{1}{2} kx^2$
$E_p = \frac{1}{2} (52.8)(0.50)^2$
$E_p = 6.6 J$

...compared to how much kinetic energy the car started with...

$E_k = ½ mv^2$
$E_k = ½ (0.212) (12)^2$
$E_k = 15.264 J = 15 J$

So there was 15.264 J – 6.6 J = 8.664 J = 9 J of energy “lost”. This was probably released as thermal energy as the spring bent.

Example 4: A pendulum is pulled aside and then released as shown in the diagram. **Determine** its speed at the bottom of the swing.

Even though this may look like a difficult problem, it really isn’t. All we need to do is keep in mind that energy is conserved. The pendulum bob is 13.0cm above a reference point (it doesn’t matter if it swings to get there), so it has potential energy. When it gets to the bottom of its swing, all that energy will have become kinetic energy.

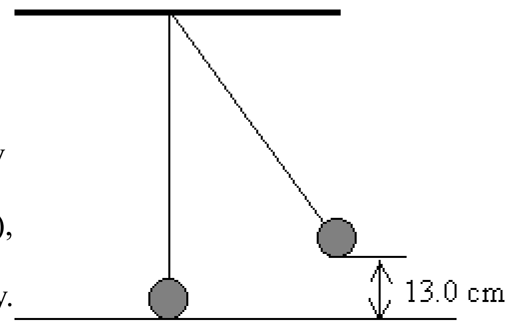

Illustration 2: Pendulum.

$E_k + E_p = E_k' + E_p'$

$E_p = E_k'$

Cancel the masses.

$mgh = ½ mv^2$

$gh = ½ v^2$

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.130)} = 1.60\,m/s$$

Example 5: A system with a frictionless pulley (known as “Atwood’s Pulley”) is shown in this diagram. If the weights are released, **determine** the speed of the 2.0 kg box as the 3.0 kg box hits the floor.

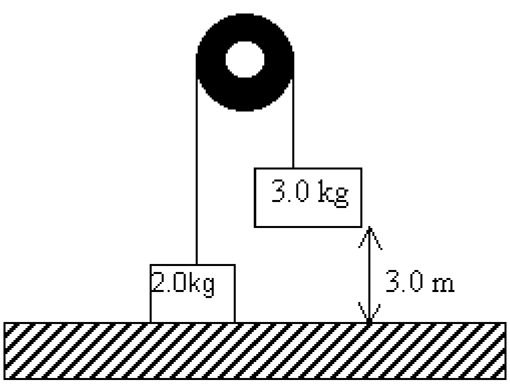

Illustration 3: Atwood's Pulley.

Again, the energy before anything starts to move must be equal to the energy at the end. Before they start to move, only one of the masses has any energy; the 3.0kg box is 3.0m in the air, so it has potential energy…

Before

$E_p = mgh = (3.0)(9.81)(3.0) = 88.29 J$

When the 3.0kg box hits the floor, the 2.0kg box will be 3.0m in the air. This is the only value for energy after that we can calculate right now.

After

E_p = mgh = (2.0)(9.81)(3.0) = 58.86 J

Which means that we are unable to account for 88.29 J – 58.86 J = 29.43 J of energy. This energy must be divided between the two masses, since they also have kinetic energy. But since they have unequal masses, the energy will be divided between them in a ratio based on their masses.

3.0 (x) + 2.0 (x) = 29.43 J

5 x = 29.43 J

x = 5.886 J

So the 2.0kg box has 2.0 (5.8J) = 11.772 J of kinetic energy. Figuring out its speed is easy now…

$$E_k = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(12)}{2}} = 3.4 m/s$$

Note: Both boxes must be traveling at the same speed. Try using the same method to calculate the speed of the 3.0kg box.