# Lesson 44: Acceleration, Velocity, and Period in SHM

Since there is a restoring force acting on objects in SHM it makes sense that the object will accelerate.

- In Physics 20 you are only required to explain this acceleration for masses on horizontal springs with no friction, and basic pendulums. Vertical springs will not be covered here.

## Acceleration of a Mass on a Spring

As a mass bounces back and forth on a spring, it will have a changing acceleration.

- The changing acceleration happens because the restoring force is always changing.
- As long as the situation is frictionless, there are no other forces to consider, so the net force will be the restoring force.

$$F_{NET}=F_s$$
$$ma=-kx$$
$$a=\frac{-kx}{m}$$

**Example 1**: Determine the acceleration of a 0.250kg mass on the end of a 54.9N/m spring if it has been...

a) stretched 12cm from its equilibrium and released.
b) compressed 25cm from its equilibrium and released.

a)
$$a=\frac{-kx}{m}$$
$$a=\frac{-54.9(0.12)}{0.250}$$
$$a=-26\,m/s^2$$

b)
$$a=\frac{-kx}{m}$$
$$a=\frac{-54.9(-0.25)}{0.250}$$
$$a=55\,m/s^2$$

These answers are the acceleration at that moment. An instant later (when they have moved to a different position) their accelerations will be different.
The answer in (a) is negative because the spring has been stretched; the mass is trying to accelerate in the opposite direction back to equilibrium. In (b) the answer is positive because the object was compressed and the mass is trying to accelerate back in the positive direction to equilibrium.

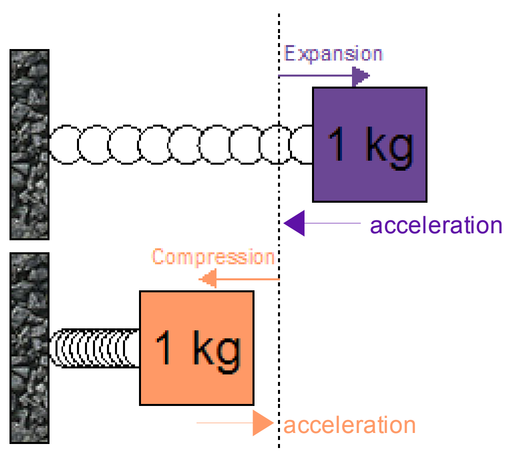


*Illustration 1: Mass undergoing acceleration.*

## Velocity of a Mass on a Spring

Some people think that a big acceleration automatically means that the object is moving at a big velocity.

- Remember that in the examples above the mass has just been released from rest, so its velocity starts at zero.
- As it accelerates back to the equilibrium point, the elastic potential energy that was stored in the spring is converted to kinetic energy as the mass goes faster and faster.
- At the equilibrium point there is no elastic potential energy remaining, so it has all become kinetic energy (this is still assuming a frictionless surface).
    - It is at the equilibrium point that the mass is moving at its maximum velocity.
- We will call the furthest that the mass is from the equilibrium point the **amplitude** (A) of the spring. It is still measured in metres.
- If we want to calculate the maximum velocity of a mass on a spring (as it passes through its equilibrium point) when it is released with a certain amplitude, we can use conservation of energy...

$$E_m = E_m'$$
$$E_k + E_p = E_k' + E_p'$$
$$E_p = E_k'$$
$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{max}^2$$
$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$
$$kA^2 = mv_{max}^2$$
$$v_{max}^2 = \frac{kA^2}{m}$$
$$v_{max} = \sqrt{\frac{kA^2}{m}}$$
$$v_{max} = A\sqrt{\frac{k}{m}}$$

$v_{max}$ = maximum velocity at equilibrium (m/s)
A = amplitude of mass (m)
k = spring constant (N/m)
m = mass (kg)

**Example 2**: A 17kg mass is pulled 13cm away from its equilibrium point, on a spring with a 367 N/m constant. **Determine** its maximum velocity as it passes through equilibrium.

$$v_{max} = A\sqrt{\frac{k}{m}}$$
$$v_{max} = 0.13\sqrt{\frac{367}{17}}$$
$$v_{max} = 0.60\, m/s$$

## Period of a Mass on a Spring

It is possible to show that SHM very closely resembles circular motion.

- We won't be going in to all the reasons here, but I'm hoping you will trust me.
- The one thing we need to keep in mind for the following manipulations to make sense is that we can consider the radius of a circle (r) to be the same as the maximum amplitude (A) of an object in SHM.
- This means we can play around with an old formula from circular motion this way...

$$v = \frac{2\pi r}{T}$$
$$v_{max} = \frac{2\pi A}{T}$$

- We can make this derived formula equal to the formula from the last section...

$$v_{max} = v_{max}$$
$$A\sqrt{\frac{k}{m}} = \frac{2\pi A}{T}$$
$$\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$
$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$

T = period (s)
m = mass (kg)
k = spring constant (N/m)

**Example 3**: Using the information from the previous example, **determine** the period of the mass.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{17}{367}}$$
$$T = 1.4\, s$$

## Period of Pendulums

One of the first people to realize how physics could explain pendulums was Galileo.

- According to Galileo's own notes, he was visiting a local church one day when he happened to look at a person lighting the lamps that were hanging in the church.
- These lanterns were hanging from the roof by long chains. The lanterns themselves were quite

heavy.

- The person lighting the lanterns was using a very long pole with a taper (fancy thin candle) on the end to light them… each time he reached up with the pole, he would nudge the lantern a bit which started it swinging.
- Galileo noticed that although all the lanterns were the same mass, the length of the chain they were dangling from seemed to change how fast they were swinging.
- In order to get some sort of measurement of the time it took them to swing, he actually used his own pulse to time them!

Although we have come a long way from the work that Galileo started, we still have a pretty simple equation that we can use to figure out how long it takes for a pendulum to swing…

$$T=2\pi\sqrt{\frac{\ell}{g}}$$

T = period of one swing (s)
ℓ = length of wire (m)
g = gravity (m/s$^2$)

Notice that the mass of the **bob** (the weight at the end of a pendulum) is not in the formula.

- This agrees with Galileo's observation that different masses on the end of the pendulum will not affect the period of the swing.
- The only thing a heavy bob does is keep the pendulum going for a longer period of time before stopping.
  - Think of it terms of inertia… a heavy bob has lots of inertia, so the pendulum swings back and forth for a long time before it stops. A lighter bob has little inertia, so it only swings for a little while before coming to rest at its equilibrium position.
- What really matters in the case of the pendulum is the length of the pendulum. If you've ever looked carefully at the pendulum on a Grandfather clock, you might have noticed that the bob can move up and down a little. This is to adjust the period of the pendulum so that the clock runs at the right speed.

Be careful when you are using this formula. Remember three things:

1. The period is the time it takes to complete ***one full swing***… that means if you let go of the bob (the weight on the end), it will swing away from your hand and back to your hand. That's one complete swing. Back to where it started and ready to move in the original direction again.
2. This formula only works well if the pendulum is held at an angle of less than 15° from equilibrium position. As the angle gets further past 15° errors start to creep in.
3. Be careful if you need to solve it for "ℓ" or "g". You'll see in the following examples how to do this.

**Example 4**: **Determine** the period of a pendulum that is 12.5 m long.

Since we have no reason to think otherwise, we will assume that it is on Earth.

$$T=2\pi\sqrt{\frac{\ell}{g}}$$

$$T=2\pi\sqrt{\frac{12.5}{9.81}}$$

$$T=7.09\,s$$

**Warning!**
I used 3.14 for pi , which gave me three sig digs. Do not use the pi button on your calculator.

**Example 5**: We decide to measure gravity in a particular location on Earth. I use a 2.75m long pendulum and find that it has a period of 3.33 s. **Determine** the acceleration due to gravity in this area.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{4\pi^2 \ell}{T^2}$$

$$g = \frac{4\pi^2(2.75)}{3.33^2}$$

$$g = 9.78\, m/s^2$$

**Example 6**: **Determine** the length of pendulum that would have a period of one minute.

We can already make a pretty good guess that this is going to be a pretty long pendulum to take that long to swing.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\ell = \frac{g T^2}{4\pi^2}$$

$$\ell = \frac{9.81(60)^2}{4\pi^2}$$

$$\ell = 895\, m$$

Yikes! Almost one kilometre long. And before you get grumpy about sig digs, notice that in the question I said “one minute”, which is by definition exactly 60 seconds, so I had an infinite number of sig digs there. Only the measurement of the acceleration due to gravity and pi have sig digs I need to worry about (three on each).